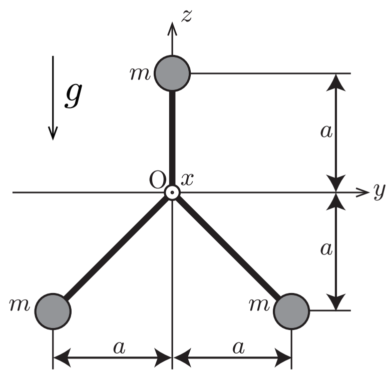
2020 ENTRANCE EXAMINATION FOR INTERNATIONAL MASTER'S PROGRAM

Departments of Mechanical Engineering and Hydrogen Energy Systems

Dynamics of Machinery (Group A) [09:00–10:30]

Examinee's number ______________________ Score ____________

I. The same three point masses of mass m are connected with massless rigid beam as shown in the figure. In the figure, all the three point masses are on the O$-yz$ plane. Set the Cartesian coordinate system O$-xyz$ so that the z axis is vertically upward, the y axis is horizontally right direction, and the x axis is perpendicular to the paper surface (with the front side of the paper being positive). Consider the small vibration around a pivot point O of this object. The gravitational acceleration is g as indicated by an arrow in the figure. Answer the following questions. (25 points)



(1) Find the center of the gravity G(x_G, y_G, z_G) of the object.

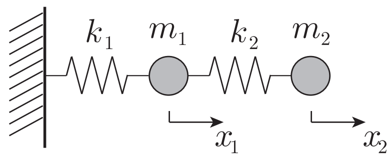
(2) Determine the potential energy U and the kinetic energy T when the object rotates θ radian in yz plane around the x axis. Let θ be a generalized coordinate, and the reference plane of the potential energy be $z = 0$.

(3) Derive an equation of motion of the object for the angle θ, and linearize the equation when the vibration is very small ($|\theta| \ll 1,\ |\dot{\theta}| \ll 1$).

(4) Find a natural period T_1 of this small vibration around the x axis.

(5) Considering the case of the same small vibration but around the y axis not around the x axis, derive a linearized equation of motion for a rotation angle ϕ around the y axis. And also find a natural period T_2 of this vibration.

II. As shown in the figure, consider a free vibration of 2DOF linear system consisting of point masses of mass m_1, m_2, and two springs of spring constant k_1, k_2. Left end of the system is fixed. Displacements of two point masses are x_1, x_2 respectively. Answer the following questions. (25 points)



(1) Using $\boldsymbol{x} = [x_1\ \ x_2]^T$ as a displacement vector, derive the equation of motion of free vibration of this system in matrix form $\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{0}$. Here $\boldsymbol{M}$ and $\boldsymbol{K}$ denote the mass and stiffness matrices respectively and $[\ ]^T$ means transpose.

(2) Derive the characteristic equation (frequency equation) of this system and solve this characteristic equation for eigenvalue ω^2.

(3) Find two natural angular frequencies ω_1, ω_2 and two natural modes $\boldsymbol{X}_1,\ \boldsymbol{X}_2$ of this system when $m_1 = m_2 = 1$kg, $k_1 = k_2 = 1$N/m. Normalize the natural modes so that the first element of each mode is equal to one.

(4) Under the condition of $m_1 = m_2 = m$, and $k_2 = ak_1$(a is a positive real constant), find the eigenvalue ω^2 using m, k_1, a.

(5) Under the same condition as in (4), and besides, $k_1 \gg k_2$, find the natural angular frequencies ω_1, ω_2 and discuss the effect of the condition of $k_1 \gg k_2$ on the motion of the system at ω_1 and ω_2 respectively.