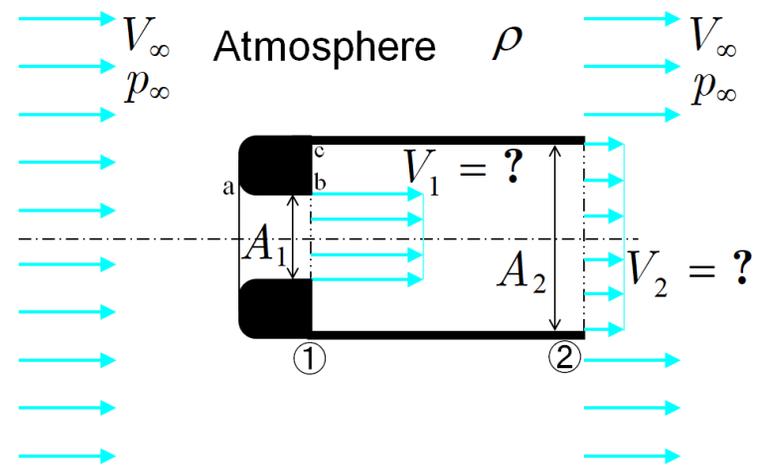


Examinee's number _____

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I . As shown in the figure, an axisymmetric duct with a sudden expansion is installed parallel to the air flow in the atmosphere that flows uniformly in the horizontal direction at a velocity of V_∞ and a pressure of p_∞ . The cross-sectional area of the duct is A_1 at the upstream of the sudden expansion, namely at the duct inlet, and A_2 at the downstream. Since the inlet at the end of the duct shown as 'a' in the figure has a smooth surface shape, the air flowing into the duct flows from the inlet to the cross section ① without loss, and then it flows from the cross section ① into the sudden expansion with a uniform velocity distribution of V_1 which is unknown. At the cross section ①, the velocity around the air flowing into the sudden expansion is negligible,



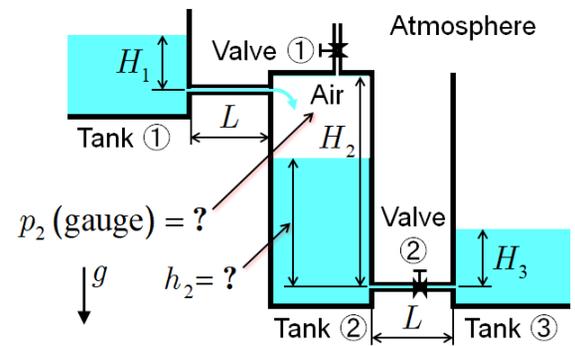
while the pressure is uniform over the entire cross section including the b-c part shown in the figure. At the duct outlet, which is the cross section ② sufficiently distant from the cross section ①, the flow becomes uniform again (the pressure is also uniform) and the air flows out to the atmosphere at an unknown velocity of V_2 . The external flow around the duct outlet is also uniform with the velocity of V_∞ and the pressure of p_∞ . Assuming that the density ρ of air is constant, that the frictional force on the inside and outside walls of the duct is negligible, and that the wall thickness at the duct outlet is sufficiently thinner than the duct cross-sectional area, answer the following questions. (25 Points)

- (1) Find the flow velocity ratio of the cross section ② and the cross section ①, V_2/V_1 .
- (2) Express the pressure difference, $p_1 - p_\infty$, between the cross section ① and the upstream of the duct using V_∞, V_1 and ρ .
- (3) Express the pressure difference, $p_1 - p_2$, between the cross section ① and the duct outlet section ② using V_1, ρ, A_1 and A_2 .
- (4) Considering that the pressure, p_2 , at the duct outlet section ② is equal to the uniform pressure, p_∞ , of the external flow around the duct, express the ratio, V_1/V_∞ , of the velocity at the cross section ① to the uniform velocity of the external flow around the duct using A_1 and A_2 .

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II. As shown in the figure, two large water tanks ① and ③, which are open to the atmosphere with an atmospheric pressure of p_a and whose water levels are kept constant, are connected by a water flow system composed of two horizontal circular-pipes and a closed tank ②. The inner diameter of d , length of L , and pipe friction factor of λ are the same for two circular-pipes. The tank ② is sealed by an upper wall having a vertical short circular-pipe with a valve ①. When the valve ① is opened, the upper part of the tank ② is open to the atmosphere. The horizontal circular-pipe between the tank ① and the tank ② has its left end connected to the lower position by H_1 from the water surface of the tank ①, and its right end connected to the upper part of the tank ② where the air is accumulated. On the other hand, the horizontal circular-pipe between the tank ② and the tank ③ has a valve ②. The left end of the latter horizontal circular-pipe is connected to the lower position by H_2 from the upper wall surface of the tank ②, and its right end is connected to the lower position by H_3 from the water surface of the tank ③. Since the tank ② is not large enough, its water level, h_2 , based on the center axis of the horizontal circular-pipe between the tanks ② and ③ and the gauge pressure of the air, p_2 , accumulated above its water surface are unknown, and change depending on the opening degree of the valves ① and ②; however, the water level, h_2 , never reaches the height level of the horizontal pipe between the tanks ① and ②. Assume that the flow loss due to the inlet of pipe is negligible, that there is no vena contracta at the outlet of pipe, that the pipe friction factor is constant without depending on the water velocity, that the velocity of water in the tank ② is also small enough to be ignored except near the inlet and outlet of the pipe, and that the density of air is so negligibly small as compared with the density of water. Suppose that the valve length is negligible compared with the pipe length, and that the flow loss due to the valve is negligible only at full opening of the valve. The density of water is denoted by ρ , and the acceleration of gravity by g . Answer the following questions. (25 Points)



- (1) When both valves ① and ② are fully opened, the water level in the tank ② becomes $h_2 > H_3$ and the flow becomes steady. Express the volumetric flow rate of water, Q , flowing into the tank ③ using, H_1 , d , L , λ and g .
- (2) In the state of the question (1) above, express the water level h_2 in the tank ② using H_1 and H_3 .
- (3) When the valve ② becomes half-closed while the valve ① is kept fully opened, the water level in the tank ② increases by Δh_2 compared to the state in the question (1) above, and the flow becomes steady again. Express the loss coefficient, ζ_2 , of the valve ② in this state using H_1 , Δh_2 , d , L and λ .
- (4) From the state of the question (1) above, when both valves ① and ② are instantly fully closed, after a while, the outflow of water from the tank ① stops and the inside of the horizontal circular-pipe between the tanks ① and ② is filled with static water. Express the gauge pressure of the air, p_2 , accumulated above the water surface of the tank ② in this state using H_1 , ρ and g .