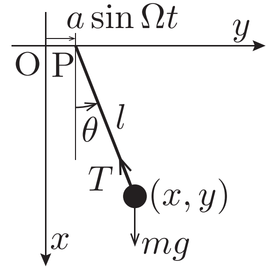
2021 ENTRANCE EXAMINATION FOR INTERNATIONAL MASTER'S PROGRAM
Departments of Mechanical Engineering and Hydrogen Energy Systems

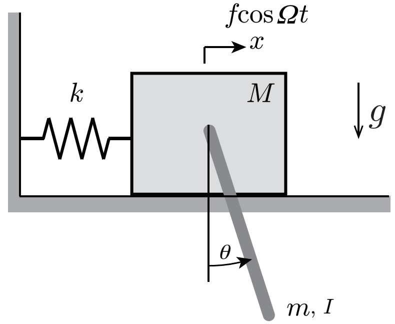
Dynamics of Machinery (Group A) [09:00–10:30]

I. As shown in the figure, consider a simple pendulum with a weight of mass m and a massless thread of length l. Define the x-axis downward in the vertical direction and the y-axis to the right in the horizontal direction. Determine the swing angle θ of the pendulum counterclockwise from the x-axis. Let the the position of the weight be (x, y). The tension of the thread is T. Consider the motion of the pendulum when the pivot point P vibrates horizontally with $a \sin \Omega t$. Assume that a is constant. The gravitational acceleration is g. Answer the following questions. (25 points)



(1) Derive the equations of motion of the weight in the x and y directions, respectively using T, θ, $\ddot{x}$ and $\ddot{y}$.
(2) Describe x and y respectively as a function of θ and t.
(3) Find the velocity and acceleration of the weight in the x and y directions.
(4) Using the results of (1) through (3), derive the equation of motion for this pendulum with θ as the generalized coordinate.
(5) When $\theta \ll 1$, linearize the equation of motion.
(6) With this linearized system and assuming that $\theta = A \sin \Omega t$, describe A as a function of Ω.

II. As shown in the figure, an object of mass M is connected to a fixed wall with a spring of spring constant k. The object is on a smooth horizontal floor. A rod of mass m and moment of inertia I is attached to the object so that the rod can rotate freely. A distance from the pivot to the center of gravity of the rod is l. A periodic force $f \cos \Omega t$ is acting on the object. Displacement of the object from the static equilibrium state is x (rightward is positive) and the rotation angle of the pendulum is θ (counterclockwise is positive). The gravitational acceleration is g. The equations of motion of this system are expressed as follows.



$$\left.\begin{array}{l}(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta + kx = f\cos\Omega t \\ ml\ddot{x}\cos\theta + (I + ml^2)\ddot{\theta} + mgl\sin\theta = 0\end{array}\right\}$$

Assuming that $M = m,\ I = ml^2,\ k = mg/l$, answer the following questions.(25 points)

(1) When the vibration is very small ($|\theta| \ll 1,\ |\dot{\theta}| \ll 1$), the linearized equation of motion of the system can be written as follows,

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f}\cos\Omega t.$$

Here the displacement vector is written as $\boldsymbol{x} = [x\ \theta]^T$ and $[\ ]^T$ means transpose. Find a mass matrix $\boldsymbol{M}$ and a stiffness matrix $\boldsymbol{K}$ of this sytem, and external force vector $\boldsymbol{f}$ as well.
(2) Find first and second natural angular frequencies $\omega_1,\ \omega_2$ ($\omega_1 < \omega_2$).
(3) Find two natural modes $\boldsymbol{X}_1,\ \boldsymbol{X}_2$, and modal matrix $\boldsymbol{\Phi}$.
(4) Describe the equations of motion using modal coordinates $\boldsymbol{\xi} = [\xi_1\ \ \xi_2]^T$.
(5) Calculate the solution of the free vibration of this system under the initial condition of $\boldsymbol{x}|_{t=0} = [x_0\ 0]^T$.
(6) Calculate the solution of the forced vibration of this system.