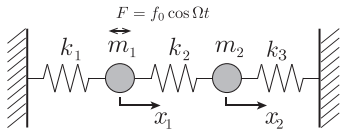
2022 ENTRANCE EXAMINATION FOR INTERNATIONAL MASTER'S PROGRAM
Departments of Mechanical Engineering and Hydrogen Energy Systems

Dynamics of Machinery (Group A) [09:00–10:30]

I.As shown in the figure, consider a 2DOF linear forced vibration system consisting of two point masses of mass m_1, m_2, and three springs of spring constant k_1, k_2 and k_3. Both ends of the system are fixed, and a force $F = f_0 \cos \Omega t$ is acting on the left point mass of m_1. All springs are at their natural lengths at the equilibrium position of the system. Answer the following questions, assuming that the solutions of free vibration of this system can be negligible. (25 points)

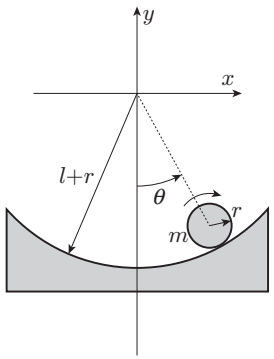


(1) Derive the equations of motion of this system.
(2) Explain the reason that the displacements x_1 and x_2, solutions of this forced vibration system, can be written as $x_1 = X_1 \cos \Omega t$ and $x_2 = X_2 \cos \Omega t$ respectively. Where X_1 and X_2 are real constants.
(3) Find the condition for the displacement x_1 to be zero regardless of the value of F.

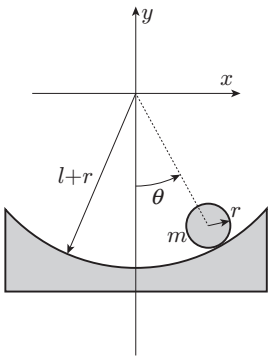
II.Consider the motion of a disk of radius r and mass m (moment of inertia around the center $I = mr^2/2$) in the vertical plane, under the following three conditions. The gravitational acceleration is g. (25 points)

Condition a: When the disk rolls along the inner surface of a cylinder of radius $l + r$ without slipping. (see Figure a)
Condition b: When the disk slides along a smooth inner surface of a cylinder of radius $l + r$ without rolling (see Figure b)
Condition c: When one end of a rigid rod (length l) of negligible mass is fixed to the center of the disk and the other end is supported freely for rotation (see Figure c)

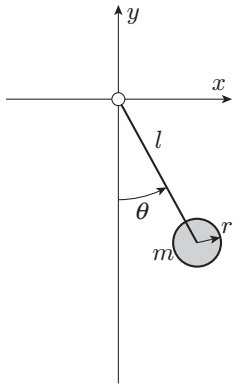
(1) For each condition, find the equation of motion for θ.
(2) Assuming that θ is small, linearize each equation of motion and find the natural angular frequencies respectively.
(3) When $0 < r < l$ as shown in the figure, show the magnitude relationship of three natural angular frequencies obtained above and briefly describe the reasons for the difference.



(a)



(b)



(c)