◇ ENTRANCE EXAMINATION FOR INTERNATIONAL MASTER'S PROGRAM 2022

Group : Mathematics [13:40∼14:40]

Question I Answer the following questions. (30 points)

(1) Find the length of the following curve within the given domain.
$y = \frac{3}{2}\left(e^{\frac{x}{3}} + e^{-\frac{x}{3}}\right) \quad (0 \leq x \leq 1)$

(2) Find the whole length of the following closed-curve.
$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 3^{\frac{2}{3}}$

(3-1) Prove that the length of a curve in the 2D polar coordinate system $r = f(\theta)$ $(0 \leq \theta \leq 2\pi)$ is given as $L = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$, where r and θ denote the radius and angle components of a curve in the 2D polar coordinate system, respectively.

(3-2) Find the whole length of the following closed-curve.
$r = 2(1 + \cos\theta) \quad (0 \leq \theta \leq 2\pi)$

Question II For a given matrix $A = \begin{bmatrix} a & a & -a \\ a & b & a \\ -a & a & b \end{bmatrix}$, answer the following questions. (40 points)

(1) Find the conditions such that the rank of the matrix A is three.

(2) Assume $a = 1$, $b = 3$ in the following questions.

(2-1) Find all the eigenvalues of the matrix A and each eigenvector whose norm is 1.

(2-2) A can be diagonalized with $S^\top AS$ using an orthogonal matrix S. Obtain the matrix S and diagonalized matrix $B = S^\top AS$.

(2-3) Calculate A^n.

Question III Find the general solution for the following differential equations. (30 points)

(1) $2y^2 - x\frac{\mathrm{d}y}{\mathrm{d}x} - 2 = 0$

(2) $(2xy - \cos x)\,\mathrm{d}x = \left(2y - x^2\right)\mathrm{d}y$

(3) $y^2 + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = x$