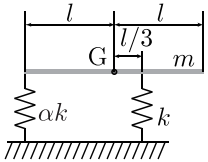
ENTRANCE EXAMINATION FOR INTERNATIONAL MASTER'S PROGRAM 2022

Group A
Dynamics of machinery

Question I. An object of mass m moves along a straight line on a smooth horizontal surface. The displacement of the object is x, and its velocity is $v(=\dot{x})$. A resistance force $-kv^2$ proportional to the square of the object's velocity acts on the object from the surrounding fluid. Here, constant k is a positive real. The displacement and velocity of the object at time $t = 0$ are $x = 0$ and $v = v_0$, respectively. Answer the following questions about the motion of this object. (25 points)

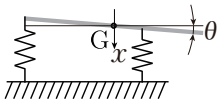
(1) Find the equation of motion of the object.
(2) Find v, the velocity of the object, as a function of time t.
(3) Find x, the displacement of the object, as a function of t.
(4) Find the time when the velocity of the object becomes $v_0/2$.
(5) Find the distance the object has traveled while the velocity of the object changes from v_0 to $v_0/2$.

Question II. Figure (a) shows a uniform rigid rod of mass m and length $2l$ (moment of inertia around center of gravity G is $ml^2/3$) is on a smooth horizontal surface. The rigid rod is connected to the rigid wall by two springs with spring constants k and αk as shown in the figure. Let the displacement of the center of gravity of the rod be x and the clockwise rotational angle around the center of gravity be θ as shown in Fig. (b) . Assuming that both x and θ are small, answer the following questions.(25 points)



(a)

(1) Find a mass matrix $\boldsymbol{M}$ and a stiffness matrix $\boldsymbol{K}$ of this system, when the displacement vector is written as $\boldsymbol{x} = [x\ \theta]^T$ and the equation of motion of the system is written as $\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{0}$. Here $[\ ]^T$ means transpose.
(2) Let $\omega_1 < \omega_2$ be two natural angular frequencies of this system. Find ω_1 and ω_2 under the condition $\alpha > 1/3$.
(3) Derive two natural modes $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ correspond to ω_1 and ω_2 obtained (2).
(4) When $\alpha = 0$, explain natural angular frequencies and natural modes of the system briefly.



(b)