(I) Answer the following questions. (40 points)

- (1) Answer the rank of the following matrix A.
  - $A = \begin{bmatrix} 1 & 1 & x \\ 1 & x & x \\ x & x & x \end{bmatrix}$
- (2-1) When the following matrix B, show the eigen equation of B as  $f_B(\lambda)$  where  $\lambda$  is eigenvalue.
  - $B = \begin{bmatrix} 2 & 0 & 0 \\ -2 & 3 & -1 \\ 1 & 0 & -1 \end{bmatrix}$
- (2-2) When  $g(\lambda) = \lambda^4 6\lambda^3 + 9\lambda^2 + 3\lambda 10$ , factorize the  $g(\lambda)$  using the  $f_B(\lambda)$ .
- (2-3) Calculate g(B).

(II) Find the general solution for the following differential equations. (30 points)

(1) 
$$\frac{dy}{dx} = \frac{x^2 + 6xy + 3y^2}{3x^2 + 2xy}$$
  
(2) 
$$\frac{dy}{dx} = \frac{x^2 - 6xy + 3y^2}{3x^2 - 6xy + y}$$
  
(3) 
$$\frac{d^2y}{dx^2} = \frac{x^2 + 6xy + 8y^2}{x + 2y}$$

(III) Answer the following questions. (30 points)

(1) Show the area surrounded by a closed curve  $\begin{cases} x = a \cos t \\ y = b \sin t \end{cases}$ ,  $(0 \le t \le 2\pi)$  is  $ab\pi$ , where a and b are a positive real number each and t is a parameter.

(2) Find the area surrounded by a curve 
$$\begin{cases} x = a (t - \sin t) \\ y = a (1 - \cos t) \end{cases}$$
,  $(0 \le t \le 2\pi)$  and x-axis, where a is a positive real number and t is a parameter.

(3) Find the area surrounded by a closed curve  $r = a(1 + \cos \theta)$ ,  $(0 \le \theta \le 2\pi)$ , where a is a positive real number and  $(r, \theta)$  is a polar coordinate system.