(I) Answer the following questions. (40 points)
(1) Answer the rank of the following matrix $A$.

$$
A=\left[\begin{array}{lll}
1 & 1 & x \\
1 & x & x \\
x & x & x
\end{array}\right]
$$

(2-1) When the following matrix $B$, show the eigen equation of $B$ as $f_{B}(\lambda)$ where $\lambda$ is eigenvalue.

$$
B=\left[\begin{array}{ccc}
2 & 0 & 0 \\
-2 & 3 & -1 \\
1 & 0 & -1
\end{array}\right]
$$

(2-2) When $g(\lambda)=\lambda^{4}-6 \lambda^{3}+9 \lambda^{2}+3 \lambda-10$, factorize the $g(\lambda)$ using the $f_{B}(\lambda)$.
(2-3) Calculate $g(B)$.
(II) Find the general solution for the following differential equations. (30 points)
(1) $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{x^{2}+6 x y+3 y^{2}}{3 x^{2}+2 x y}$
(2) $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{x^{2}-6 x y+3 y^{2}}{3 x^{2}-6 x y+y}$
(3) $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=\frac{x^{2}+6 x y+8 y^{2}}{x+2 y}$
(III) Answer the following questions. (30 points)
(1) Show the area surrounded by a closed curve $\left\{\begin{array}{l}x=a \cos t \\ y=b \sin t\end{array} \quad, \quad(0 \leq t \leq 2 \pi)\right.$ is $a b \pi$, where $a$ and $b$ are a positive real number each and $t$ is a parameter.
(2) Find the area surrounded by a curve $\left\{\begin{array}{l}x=a(t-\sin t) \\ y=a(1-\cos t)\end{array} \quad, \quad(0 \leq t \leq 2 \pi)\right.$ and $x$-axis, where $a$ is a positive real number and $t$ is a parameter.
(3) Find the area surrounded by a closed curve $r=a(1+\cos \theta),(0 \leq \theta \leq 2 \pi)$, where $a$ is a positive real number and $(r, \theta)$ is a polar coordinate system.

