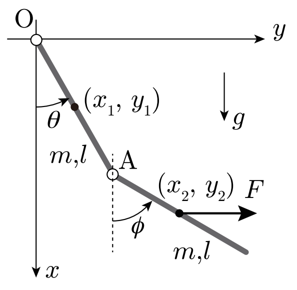
2023 ENTRANCE EXAMINATION FOR INTERNATIONAL MASTER'S PROGRAM
Departments of Mechanical Engineering and Hydrogen Energy Systems

Dynamics of Machinery (Group A) [09:00–10:30]

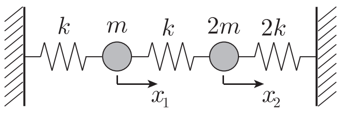
Examinee's number ______________________ Score ____________

I. As shown in the figure, a double pendulum, which consists of two same uniform bars of length l and mass m connected at point A, is suspended from point O. The motions of the two bars are in the vertical plane and they are free to rotate around points O and A. The center of gravity of the upper bar is (x_1, y_1) and that of the lower bar is (x_2, y_2). Find the angles θ and ϕ when the double pendulum is at rest with a constant external force F acting horizontally on the center of gravity of the lower bar, according to the following questions. The gravitational acceleration is g, as shown in the figure. (25 points)



(1) Find two centers of gravity of both upper and lower bars (x_1, y_1) and (x_2, y_2) respectively using l, θ and ϕ.
(2) Find virtual displacements for centers of gravity of both upper and lower bars $(\delta x_1, \delta y_1)$ and $(\delta x_2, \delta y_2)$ using $\delta\theta$ and $\delta\phi$. $\delta\theta$ and $\delta\phi$ are the virtual (angular) displacements of θ and ϕ, respectively.
(3) Calculate a virtual work δW done by both gravitational force mg and horizontal force F.
(4) Find the tangent of θ and the tangent of ϕ using the principle of the virtual work.

II. As shown in the figure, consider a 2DOF linear vibration system consisting of two point masses of mass m, $2m$, and three springs of spring constant k, k and $2k$. Both ends of the system are fixed, and all springs are at their natural lengths at the equilibrium position of the system. Answer the following questions. (25 points)



(1) Derive the equations of motion of this system.
(2) Express the equations of motion of this system in matrix form $\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{0}$. Here $\boldsymbol{M}$ and $\boldsymbol{K}$ denote the mass and the stiffness matrices, $\boldsymbol{x} = [x_1 \quad x_2]^{\mathrm{T}}$ is displacement vector and $[\ ]^{\mathrm{T}}$ means transpose.
(3) Find the frequency equation (characteristic equation) of this system.
(4) Find the eigenvalues $\omega_i^2, (i = 1, 2)$ and natural modes $\boldsymbol{X}_i, (i = 1, 2)$ of this system. Normalize the natural modes as $\boldsymbol{X}_i = [1 \quad X]^{\mathrm{T}}$.
(5) Express the general solution of free vibration of this system using ω_1 and ω_2.
(6) Calculate the free vibration of this system with the initial conditions of $x_1|_{t=0} = x_2|_{t=0} = 1$ and $\dot{x}_1|_{t=0} = \dot{x}_2|_{t=0} = 0$.