2023 ENTRANCE EXAMINATION FOR INTERNATIONAL MASTER'S PROGRAM
Departments of Mechanical Engineering and Hydrogen Energy Systems

Mathematics No.1 of 3 [13:40 $\sim$ 14:40]

Examinee's number ____________________

Score ________

I. Answer the following questions for matrix $\boldsymbol{A}$ given below. (40 points).

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1+i \\ -1+i & -i \end{bmatrix}$$

I-1) Find the eigenvalues λ_1 and λ_2 of $\boldsymbol{A}$ and the corresponding eigenvectors $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$.

I-2) Show that the eigenvectors $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are orthogonal.

I-3) Find the regular matrix $\boldsymbol{P}$ that $\boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P}$ is a diagonal matrix.

Answer below the line

Mathematics No.2 of 3 [13:40 $\sim$ 14:40]

Examinee's number

Score

II. Answer the following questions (30 points).

II-1) Solve the following initial value problem of the ordinary differential equation for $y(t)$ under the initial condition that $y(0) = 0$ and $\dfrac{\mathrm{d}y(0)}{\mathrm{d}t} = 0$.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - 2\frac{\mathrm{d}y}{\mathrm{d}t} + y = e^{2t}$$

II-2) Solve the following initial value problem of the simultaneous ordinary differential equations for each $x(t)$, $y(t)$ and $z(t)$ under the initial condition that $x(0) = \dfrac{4}{3}$, $y(0) = 0$, $z(0) = \dfrac{2}{3}$ and $\dfrac{\mathrm{d}y(0)}{\mathrm{d}t} = 2$.

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = -2y \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = x + 4y + z \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = -y \end{cases}$$

Answer below the line

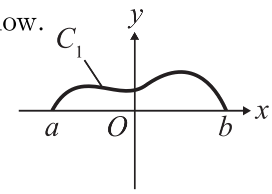
Mathematics No.3 of 3 [13:40 $\sim$ 14:40]

Examinee's number ____________________

Score __________

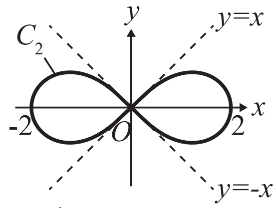
III. Answer the following questions. (30 points)

III-1) Find the surface area of volume obtained by rotating a curve C_1 : $y = f(x)$ $(a \leq x \leq b)$ around x-axis. Here, $f(x)$ is C^1-function, $f(a) = f(b) = 0$ and always positive in $a < x < b$.
Note that smooth surface area S which is parameterized by (u, v) is defined as below.



$$S = \iint_D \sqrt{\left(\frac{\partial(x,y)}{\partial(u,v)}\right)^2 + \left(\frac{\partial(y,z)}{\partial(u,v)}\right)^2 + \left(\frac{\partial(z,x)}{\partial(u,v)}\right)^2} \mathrm{d}u\mathrm{d}v,$$

where $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$, $\dfrac{\partial(y,z)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \end{vmatrix}$, $\dfrac{\partial(z,x)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} \\ \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \end{vmatrix}$.



III-2) Answer the following questions for a curve C_2 : $(x^2 + y^2)^2 = 4(x^2 - y^2)$.

III-2-1) Express the curve C_2 in the polar coordinates (r, θ).

III-2-2) Find the surface area of volume obtained by rotating the curve C_2 around x-axis.

Answer below the line