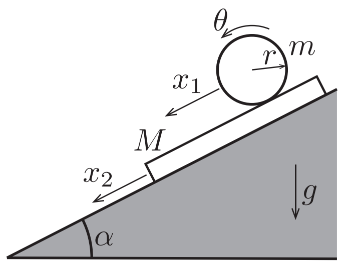
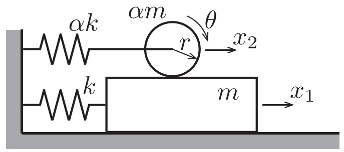
ENTRANCE EXAMINATION FOR INTERNATIONAL MASTER'S PROGRAM 2023
Departments of Mechanical Engineering and Hydrogen Energy Systems

Dynamics of Machinery (Group A) [09:00–10:30]



I.As shown in the figure, a cylinder of mass m and radius r (moment of inertia around the center $I = mr^2/2$) is on a plate of mass M. The plate is on a smooth slope with angle α. The plate slides on the slope without friction, and the cylinder rolls without slipping on the plate. A displacement along the slope and a rotational angle of the cylinder are x_1 and θ respectively. A displacement of the plate is x_2. Assume that the frictional force between the cylinder and the plate is F. The gravitational acceleration is g. Answer the following questions. (25 points)

(1) Derive the equations of motion of the plate and the cylinder.
(2) Find the acceleration and the angular acceleration of the cylinder and also find the acceleration of the plate.



II.As shown in the figure, a cylinder of mass αm and radius r (moment of inertia around the center $I = \alpha mr^2/2$) is on a plate of mass m. The plate is on a smooth horizontal floor. The cylinder rolls without slipping on the plate, and the plate slides on the floor without friction. A displacement of the plate is x_1. A displacement and a rotational angle of the cylinder are x_2 and θ respectively. The cylinder is connected to the wall with a linear spring of stiffness αk. The plate is also connected to the wall with a linear spring of stiffness k. α is a positive constant. Answer the following questions. (25 points)

(1) Determine the kinetic energy T and the potential energy U of this system.
(2) Derive the Lagrange's equations of motion of the system using x_1 and x_2 as the generalized coordinates.
(3) Express the equations of motion of this system in matrix form $\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{0}$. Here $\boldsymbol{M}$ and $\boldsymbol{K}$ denote the mass and the stiffness matrices, $\boldsymbol{x} = [x_1 \quad x_2]^{\mathrm{T}}$ is generalized displacement vector and $[\ ]^{\mathrm{T}}$ means transpose.
(4) Find the frequency equation (characteristic equation) of this system.
(5) Find the eigenvalues $\omega_i^2, (i = 1, 2)$ and natural modes $\boldsymbol{X}_i, (i = 1, 2)$ of this system. Normalize the natural modes as $\boldsymbol{X}_i = [1 \quad X]^{\mathrm{T}}$.