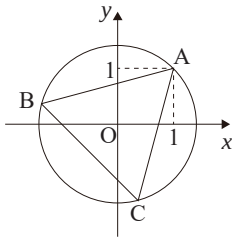
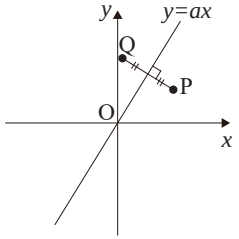
(I) Answer the following questions about geometric transformation. (30 points)

(1) An equilateral triangle inscribed a circle $x^2 + y^2 = 2$ is set so that one of its apex A is on $(1, 1)$ as shown in the figure. Answer the coordinate values of the remaining apexes of the equilateral triangle, B and C.




(2) Linear transformation matrix $M$ represents mirroring transformation against a line $y = ax$. The transformation $M$ mirrors the point P to the point Q as in the figure. Then, express $M$ using $a$.



(II) Find the general solution for the following differential equations. (35 points)

(1) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6x - 2y - 3}{3x - y + 2}$

(2) $x^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} = y^2 - xy - 3x^2$

(3) $\dfrac{\mathrm{d}^3 y}{\mathrm{d}x^3} - 3\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2y = 10 \sin x$

(III) Consider a volume $V$ which is the intersection (common part) of a sphere $A$ and a cylinder $B$. Answer the following questions. (35 points)

(1) If sphere $A_1$: $x^2 + y^2 + z^2 \leqq 4a^2 \quad (a > 0)$ and cylinder $B_1 : x^2 + y^2 \leqq a^2 \quad (a > 0)$, find the volume $V_1$.

(2) If sphere $A_2$: $x^2 + y^2 + z^2 \leqq a^2 \quad (a > 0)$ and cylinder $B_2 : x^2 + y^2 \leqq ax \quad (a > 0)$, find the volume $V_2$.

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☐ Applicant of INTERNATIONAL MASTER'S PROGRAM should answer in English.