♢ ENTRANCE EXAMINATION FOR INTERNATIONAL MASTER'S PROGRAM 2024

Group : Mathematics [13:15∼14:15] Number _____

Applicant of INTERNATIONAL MASTER'S PROGRAM should answer in English.

(I) Answer the following questions for matrix $\boldsymbol{A}$. (40 points)

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

(1) Find all eigenvalues of matrix $\boldsymbol{A}$ and their corresponding eigenvectors.

(2) Find a matrix $\boldsymbol{P}$ satisfying the following condition while $a > b > c$.

$$\boldsymbol{P^{-1}AP} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

(3) Consider a set of differential equations expressed as $\boldsymbol{y'} = \boldsymbol{Ay}$, with $\boldsymbol{y}$, $\boldsymbol{y'}$ and $\boldsymbol{z}$ defined as follows. y_1, y_2 and y_3 are each functions of x. Express z_1, z_2 and z_3 satisfying $\boldsymbol{y} = \boldsymbol{Pz}$ using x.

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \quad \boldsymbol{y'} = \begin{bmatrix} \frac{dy_1}{dx} \\ \frac{dy_2}{dx} \\ \frac{dy_3}{dx} \end{bmatrix}, \quad \boldsymbol{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

(4) Find the solution of the simultaneous differential equations $\boldsymbol{y}$ while $y_1 = 0$, $y_2 = -4$ and $y_3 = 7$ when $x = 0$.

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(II) Find the general solution for the following differential equations. (30 points)

(1) $x^2 + 2xy\dfrac{\mathrm{d}y}{\mathrm{d}x} - y^2 = 0$

(2) $\dfrac{\mathrm{d}y}{\mathrm{d}x} + xy = x$

(3) $(2x + \mathrm{e}^y)\,\mathrm{d}x + (x\mathrm{e}^y + \sin y + y\cos y)\,\mathrm{d}y = 0$

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(III) Answer the following questions. (30 points)

(1) Illustrate the region $D = \{(x, y) \mid x^2 < y < x\}$ in the $x - y$ plane.

(2) Find $I = \iint_D 7x^3y^2 dxdy$.