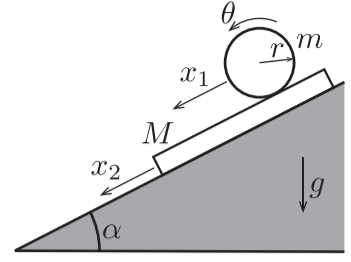


Examinee's number _____

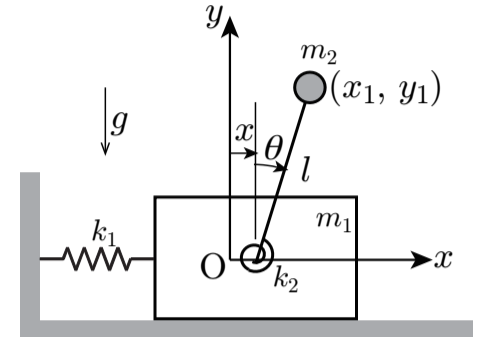
Score _____

I. As shown in the figure, a cylinder of mass m and radius r (moment of inertia around the center $I = mr^2/2$) is on a plate of mass M . The plate is on a smooth slope with angle α . The plate slides on the slope without friction, and the cylinder rolls without slipping on the plate. A displacement along the slope and a rotational angle of the cylinder are x_1 and θ respectively. A displacement of the plate is x_2 . Assume that the frictional force between the cylinder and the plate is F . The gravitational acceleration is g . Answer the following questions. (25 points)



- (1) Derive the equation of motion of the plate.
- (2) Derive the equations of motion, both translational and rotational, of the cylinder.
- (3) Find the acceleration and the angular acceleration of the cylinder and also find the acceleration of the plate.

II. As shown in the figure, an object of mass m_1 moves on a smooth horizontal floor and the object is connected with a linear spring of spring constant k_1 to a vertical wall. A mass point m_2 is fixed to one end of a massless rigid beam of length l and the other end of the beam is jointed to the object with a rotating spring k_2 . A relation between a torque τ of the rotating spring and a rotating angle θ is $\tau = k_2\theta$. A static orthogonal coordinate $O-xy$ is set as shown in the figure. A coordinate origin O is set on the beam end when the linear spring k_1 is at its natural length. Let a horizontal displacement of the object be x and a rotating angle of the beam be θ as shown in the figure. Answer the following questions using x and θ as the generalized coordinates. The gravitational acceleration is g . (25 points)



- (1) Determine the kinetic energy T and the potential energy U of this system.
- (2) Derive the Lagrange's equations of motion of the system.
- (3) Linearize the equations of motion and express them in matrix form $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}$ when $|x|$, $|\dot{x}|$, $|\theta|$ and $|\dot{\theta}|$ are small. Here \mathbf{M} and \mathbf{K} denote the mass and stiffness matrices respectively, $\mathbf{x} = [x \ \theta]^T$ is generalized displacement vector and $[]^T$ means transpose.
- (4) Find the characteristic equation (frequency equation) of this linearized system when $k_2 = \alpha k_1 l^2$, $m_1 = \beta m_2$, and $k_1/m_2 = g/l = \gamma$. Where α , β and γ are positive real.
- (5) Find the natural angular frequencies ω_i , ($i = 1, 2$) and natural modes \mathbf{X}_i , ($i = 1, 2$) of this linear system when $\alpha = 2$, $\beta = 3/2$. Normalize the natural modes as $\mathbf{X}_i = [1 \ X]^T$.