Thermal Engineering (Group A) [10:45~12:15]

Examinee's number ______________________________ Score ________

(I) Two chambers, A and B, are separated by a movable insulating wall as shown in Fig. 1.1. Although the wall seals the gas, it can move smoothly without any friction. Chambers A and B contain the same type of ideal gas, whose specific heat ratio κ is independent of temperature. At State 1, pressure, temperature and volume of the gas in Chamber A are p_1, T_1 and V_{A1}, respectively. Pressure, temperature and volume of the gas in Chamber B are p_1, T_1 and V_{B1}, respectively. Now the gas in Chamber A starts to be heated, and simultaneously the gas in Chamber B starts to be cooled, causing the movable wall to shift to the right. This process is quasi-static, and the heating rate of Chamber A and the cooling rate of Chamber B are equal. At the moment when the total amount of heat Q_{12} is added to Chamber A, i.e., at the moment when the total amount of heat Q_{12} is removed from Chamber B, heating of Chamber A and cooling of Chamber B are stopped simultaneously. This is State 2 as shown in Fig. 1.2. At State 2, pressure, temperature and volume of the gas in Chamber A are p_2, T_{A2} and V_{A2}, respectively. Pressure, temperature and volume of the gas in Chamber B are p_2, T_{B2} and V_{B2}, respectively. Assuming that the heat capacity of the chambers can be considered negligible, answer the following questions. (25 points)

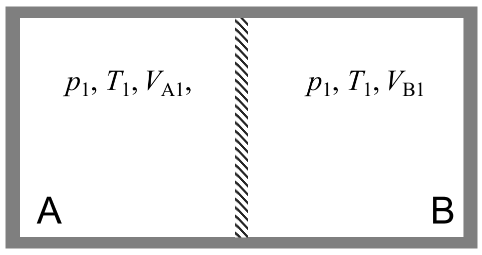

Fig. 1.1. State 1

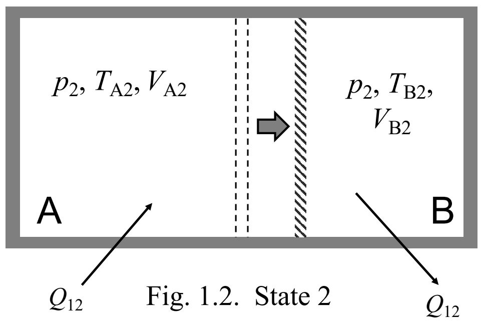

Fig. 1.2. State 2

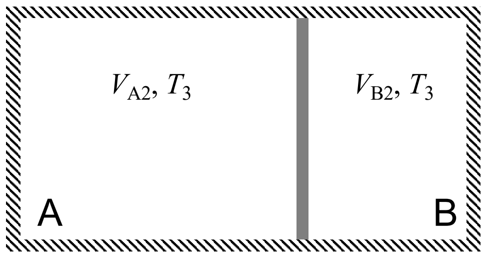

Fig. 1.3. State 3

(1) Prove that the process from State 1 to State 2 is a constant-pressure process. In other words, prove that pressure p of each chamber at an arbitrary state between State 1 and State 2 is equal to p_1. In the answer, define temperature and volume of the gas in Chamber A at an arbitrary state as T_A and V_A, respectively. In the same way, define temperature and volume of the gas in Chamber B as T_B and V_B, respectively.

You may continue answering the following questions on the premise that the proposition in Question (1) is true whether you prove it or not.

(2) Determine T_{A2} and T_{B2} in terms of p_1, V_{A1}, V_{B1}, Q_{12}, T_1 and κ as necessary.

(3) Determine the changes of internal energy of Chambers A $U_{A2}-U_{A1}$ and B $U_{B2}-U_{B1}$ during the process from State 1 to State 2 in terms of Q_{12} and κ.

(4) Determine the work L_{A12} done by the gas in Chamber A on the gas in Chamber B during the process from State 1 to State 2 in terms of Q_{12} and κ.

(5) Determine the change of entropy of total gas $\Delta S_A+\Delta S_B$ during the process from State 1 to State 2 in terms of p_1, V_{A1}, V_{B1}, Q_{12}, T_1 and κ.

(6) While the entire chamber is thermally insulated from the outside, the wall separating Chambers A and B is fixed in position and made thermally transparent. After thermal equilibrium is established, the temperatures of the gases in both chambers become T_3. This is State 3 as shown in Fig. 1.3. Determine T_3 in terms of the physical quantities given in the problem.

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(II) Answer the following questions using only the physical quantities provided in the figures unless otherwise specified. (25 points)

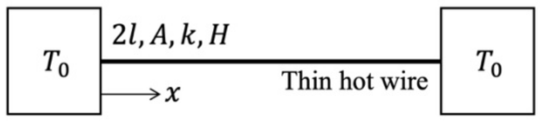

Fig. 2.1

A thin wire of length $2l$, cross-sectional area A, and thermal conductivity k is uniformly heated by volumetric heat generation H per unit volume and time. Both ends of the wire are in contact with the heat sinks and maintained at temperature T_0 as shown in Fig. 2.1. The thin wire is placed in a vacuum, where both convective and radiative heat transfer can be neglected.

(1) Assuming one-dimensional steady-state heat conduction in the x direction, the temperature distribution of the wire is given by $T(x) = -\frac{H}{2k}x^2 + \frac{Hl}{k}x + T_0$. Determine the average temperature T_{ave} of the wire.

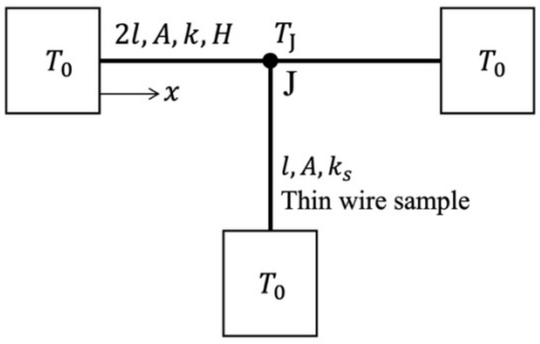

Fig. 2.2

A thin wire sample of length l, cross-sectional area A, and thermal conductivity k_s is placed in contact with the center of the hot wire as shown in Fig. 2.2. The temperature at the junction J becomes T_J, while the other end of the wire sample is maintained at a constant temperature T_0. The contact thermal resistance between the hot wire and the thin wire sample at the junction J is negligible.

(2) Does the average temperature of the hot wire increase or decrease compared to that in Question (1)?

(3) By solving the one-dimensional heat conduction equation in the x direction, $k\frac{d^2T}{dx^2} + H = 0$, obtain the temperature distribution $T(x)$ (where $0 \le x \le l$) in the left half of the hot wire. Then, determine the average temperature T'_{ave} of the hot wire.

(4) Determine the temperature gradient on the hot wire at the junction J and the amount of heat transfer $\dot{Q}_s$ flowing into the thin wire sample by applying Fourier's law to the junction J on the hot wire. Note that the system is symmetric, and equal amounts of heat flow into the wire sample from both sides of the hot wire.

(5) Determine $\dot{Q}_s$ in a different form from that in Question (4) by representing the amount of heat transfer using the thermal resistance of the thin wire sample.

(6) Let ΔT_{ave} be the difference between the average temperatures of the hot wires in Figs. 2.1 and 2.2; that is $\Delta T_{ave} = T_{ave} - T'_{ave}$. Determine an expression for the thermal conductivity k_s of the thin wire sample in terms of ΔT_{ave} and the physical quantities in the figures except T_J.

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